A.7 Errors and the Algebra of Calculus

- **Avoid common algebraic errors.**
- **Recognize and use algebraic techniques that are common in calculus.**

Algebraic Errors to Avoid

This section contains five lists of common algebraic errors: errors involving parentheses, errors involving fractions, errors involving exponents, errors involving radicals, and errors involving dividing out. Many of these errors are made because they seem to be the *easiest* things to do. For instance, the operations of subtraction and division are often believed to be commutative and associative. The following examples illustrate the fact that subtraction and division are neither commutative nor associative.

Not commutative	**Not associative**
$4 - 3 \neq 3 - 4$	$8 - (6 - 2) \neq (8 - 6) - 2$
$15 \div 5 \neq 5 \div 15$	$20 \div (4 \div 2) \neq (20 \div 4) \div 2$

Errors Involving Parentheses

Potential Error	Correct Form	Comment
$\cancel{a - (x - b) = a - x - b}$	$a - (x - b) = a - x + b$	Distribute to each term in parentheses.
$\cancel{(a + b)^2 = a^2 + b^2}$	$(a + b)^2 = a^2 + 2ab + b^2$	Remember the middle term when squaring binomials.
$\cancel{\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{2}(ab)}$	$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{4}(ab) = \frac{ab}{4}$	$\frac{1}{2}$ occurs twice as a factor.
$\cancel{(3x + 6)^2 = 3(x + 2)^2}$	$(3x + 6)^2 = [3(x + 2)]^2$ $= 3^2(x + 2)^2$	When factoring, raise all factors to the power.

Errors Involving Fractions

Potential Error	Correct Form	Comment
$\cancel{\frac{2}{x + 4} = \frac{2}{x} + \frac{2}{4}}$	Leave as $\frac{2}{x + 4}$.	The fraction is already in simplest form.
$\cancel{\frac{\left(\frac{x}{a}\right)}{b} = \frac{bx}{a}}$	$\frac{\left(\frac{x}{a}\right)}{b} = \left(\frac{x}{a}\right)\left(\frac{1}{b}\right) = \frac{x}{ab}$	Multiply by the reciprocal when dividing fractions.
$\cancel{\frac{1}{a} + \frac{1}{b} = \frac{1}{a + b}}$	$\frac{1}{a} + \frac{1}{b} = \frac{b + a}{ab}$	Use the property for adding fractions.
$\cancel{\frac{1}{3x} = \frac{1}{3}x}$	$\frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x}$	Use the property for multiplying fractions.
$\cancel{(1/3)x = \frac{1}{3x}}$	$(1/3)x = \frac{1}{3} \cdot x = \frac{x}{3}$	Be careful when expressing fractions in the form $1/a$.
$\cancel{(1/x) + 2 = \frac{1}{x + 2}}$	$(1/x) + 2 = \frac{1}{x} + 2 = \frac{1 + 2x}{x}$	Be careful when expressing fractions in the form $1/a$. Be sure to find a common denominator before adding fractions.

Errors Involving Exponents

Potential Error	Correct Form	Comment
$\cancel{(x^2)^3 = x^5}$	$(x^2)^3 = x^{2 \cdot 3} = x^6$	Multiply exponents when raising a power to a power.
$\cancel{x^2 \cdot x^3 = x^6}$	$x^2 \cdot x^3 = x^{2+3} = x^5$	Add exponents when multiplying powers with like bases.
$\cancel{(2x)^3 = 2x^3}$	$(2x)^3 = 2^3x^3 = 8x^3$	Raise each factor to the power.
$\cancel{\dfrac{1}{x^2 - x^3} = x^{-2} - x^{-3}}$	Leave as $\dfrac{1}{x^2 - x^3}$.	Do not move term-by-term from denominator to numerator.

Errors Involving Radicals

Potential Error	Correct Form	Comment
$\cancel{\sqrt{5x} = 5\sqrt{x}}$	$\sqrt{5x} = \sqrt{5}\sqrt{x}$	Radicals apply to every factor inside the radical.
$\cancel{\sqrt{x^2 + a^2} = x + a}$	Leave as $\sqrt{x^2 + a^2}$.	Do not apply radicals term-by-term when adding or subtracting terms.
$\cancel{\sqrt{-x + a} = -\sqrt{x - a}}$	Leave as $\sqrt{-x + a}$.	Do not factor minus signs out of square roots.

Errors Involving Dividing Out

Potential Error	Correct Form	Comment
$\cancel{\dfrac{a + bx}{a} = 1 + bx}$	$\dfrac{a + bx}{a} = \dfrac{a}{a} + \dfrac{bx}{a} = 1 + \dfrac{b}{a}x$	Divide out common factors, not common terms.
$\cancel{\dfrac{a + ax}{a} = a + x}$	$\dfrac{a + ax}{a} = \dfrac{a(1 + x)}{a} = 1 + x$	Factor before dividing out common factors.
$\cancel{1 + \dfrac{x}{2x} = 1 + \dfrac{1}{x}}$	$1 + \dfrac{x}{2x} = 1 + \dfrac{1}{2} = \dfrac{3}{2}$	Divide out common factors.

A good way to avoid errors is to *work slowly, write neatly,* and *talk to yourself.* Each time you write a step, ask yourself why the step is algebraically legitimate. You can justify the step below because *dividing the numerator and denominator by the same nonzero number produces an equivalent fraction.*

$$\frac{2x}{6} = \frac{\cancel{2} \cdot x}{\cancel{2} \cdot 3} = \frac{x}{3}$$

EXAMPLE 1 **Describing and Correcting an Error**

Describe and correct the error. $\cancel{\dfrac{1}{2x} + \dfrac{1}{3x} = \dfrac{1}{5x}}$

Solution Use the property for adding fractions: $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{b + a}{ab}$.

$$\frac{1}{2x} + \frac{1}{3x} = \frac{3x + 2x}{6x^2} = \frac{5x}{6x^2} = \frac{5}{6x}$$

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Describe and correct the error. $\cancel{\sqrt{x^2 + 4} = x + 2}$

Some Algebra of Calculus

In calculus it is often necessary to take a simplified algebraic expression and rewrite it. See the following lists, taken from a standard calculus text.

Unusual Factoring

Expression	Useful Calculus Form	Comment
$\dfrac{5x^4}{8}$	$\dfrac{5}{8}x^4$	Write with fractional coefficient.
$\dfrac{x^2 + 3x}{-6}$	$-\dfrac{1}{6}(x^2 + 3x)$	Write with fractional coefficient.
$2x^2 - x - 3$	$2\left(x^2 - \dfrac{x}{2} - \dfrac{3}{2}\right)$	Factor out the leading coefficient.
$\dfrac{x}{2}(x + 1)^{-1/2} + (x + 1)^{1/2}$	$\dfrac{(x + 1)^{-1/2}}{2}[x + 2(x + 1)]$	Factor out the variable expression with the lesser exponent.

Writing with Negative Exponents

Expression	Useful Calculus Form	Comment
$\dfrac{9}{5x^3}$	$\dfrac{9}{5}x^{-3}$	Move the factor to the numerator and change the sign of the exponent.
$\dfrac{7}{\sqrt{2x - 3}}$	$7(2x - 3)^{-1/2}$	Move the factor to the numerator and change the sign of the exponent.

Writing a Fraction as a Sum

Expression	Useful Calculus Form	Comment
$\dfrac{x + 2x^2 + 1}{\sqrt{x}}$	$x^{1/2} + 2x^{3/2} + x^{-1/2}$	Divide each term of the numerator by $x^{1/2}$.
$\dfrac{1 + x}{x^2 + 1}$	$\dfrac{1}{x^2 + 1} + \dfrac{x}{x^2 + 1}$	Rewrite the fraction as a sum of fractions.
$\dfrac{2x}{x^2 + 2x + 1}$	$\dfrac{2x + 2 - 2}{x^2 + 2x + 1}$	Add and subtract the same term.
	$= \dfrac{2x + 2}{x^2 + 2x + 1} - \dfrac{2}{(x + 1)^2}$	Rewrite the fraction as a difference of fractions.
$\dfrac{x^2 - 2}{x + 1}$	$x - 1 - \dfrac{1}{x + 1}$	Use long division. (See Section 2.3.)
$\dfrac{x + 7}{x^2 - x - 6}$	$\dfrac{2}{x - 3} - \dfrac{1}{x + 2}$	Use the method of partial fractions. (See Section 7.4.)

Inserting Factors and Terms

Expression	Useful Calculus Form	Comment
$(2x - 1)^3$	$\frac{1}{2}(2x - 1)^3(2)$	Multiply and divide by 2.
$7x^2(4x^3 - 5)^{1/2}$	$\frac{7}{12}(4x^3 - 5)^{1/2}(12x^2)$	Multiply and divide by 12.
$\frac{4x^2}{9} - 4y^2 = 1$	$\frac{x^2}{9/4} - \frac{y^2}{1/4} = 1$	Write with fractional denominators.
$\frac{x}{x + 1}$	$\frac{x + 1 - 1}{x + 1} = 1 - \frac{1}{x + 1}$	Add and subtract the same term.

The next five examples demonstrate many of the steps in the preceding lists.

EXAMPLE 2 Factors Involving Negative Exponents

Factor $x(x + 1)^{-1/2} + (x + 1)^{1/2}$.

Solution When multiplying powers with like bases, you add exponents. When factoring, you are undoing multiplication, and so you *subtract* exponents.

$$\begin{aligned} x(x + 1)^{-1/2} + (x + 1)^{1/2} &= (x + 1)^{-1/2}[x(x + 1)^0 + (x + 1)^1] \\ &= (x + 1)^{-1/2}[x + (x + 1)] \\ &= (x + 1)^{-1/2}(2x + 1) \end{aligned}$$

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Factor $x(x - 2)^{-1/2} + 3(x - 2)^{1/2}$.

Another way to simplify the expression in Example 2 is to multiply the expression by a fractional form of 1 and then use the Distributive Property.

$$\begin{aligned} x(x + 1)^{-1/2} + (x + 1)^{1/2} &= [x(x + 1)^{-1/2} + (x + 1)^{1/2}] \cdot \frac{(x + 1)^{1/2}}{(x + 1)^{1/2}} \\ &= \frac{x(x + 1)^0 + (x + 1)^1}{(x + 1)^{1/2}} = \frac{2x + 1}{\sqrt{x + 1}} \end{aligned}$$

EXAMPLE 3 Inserting Factors in an Expression

Insert the required factor: $\frac{x + 2}{(x^2 + 4x - 3)^2} = (\quad)\frac{1}{(x^2 + 4x - 3)^2}(2x + 4)$.

Solution The expression on the right side of the equation is twice the expression on the left side. To make both sides equal, insert a factor of $\frac{1}{2}$.

$$\frac{x + 2}{(x^2 + 4x - 3)^2} = \left(\frac{1}{2}\right)\frac{1}{(x^2 + 4x - 3)^2}(2x + 4)$$

Right side is multiplied and divided by 2.

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Insert the required factor: $\frac{6x - 3}{(x^2 - x + 4)^2} = (\quad)\frac{1}{(x^2 - x + 4)^2}(2x - 1)$.

EXAMPLE 4 Rewriting Fractions

Explain why the two expressions are equivalent.

$$\frac{4x^2}{9} - 4y^2 = \frac{x^2}{9/4} - \frac{y^2}{1/4}$$

Solution To write the expression on the left side of the equation in the form given on the right side, multiply the numerator and denominator of each term by $\frac{1}{4}$.

$$\frac{4x^2}{9} - 4y^2 = \frac{4x^2}{9}\left(\frac{1/4}{1/4}\right) - 4y^2\left(\frac{1/4}{1/4}\right) = \frac{x^2}{9/4} - \frac{y^2}{1/4}$$

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Explain why the two expressions are equivalent.

$$\frac{9x^2}{16} + 25y^2 = \frac{x^2}{16/9} + \frac{y^2}{1/25}$$

EXAMPLE 5 Rewriting with Negative Exponents

Rewrite each expression using negative exponents.

a. $\dfrac{-4x}{(1-2x^2)^2}$ **b.** $\dfrac{2}{5x^3} - \dfrac{1}{\sqrt{x}} + \dfrac{3}{5(4x)^2}$

Solution

a. $$\frac{-4x}{(1-2x^2)^2} = -4x(1-2x^2)^{-2}$$

b. $$\frac{2}{5x^3} - \frac{1}{\sqrt{x}} + \frac{3}{5(4x)^2} = \frac{2}{5x^3} - \frac{1}{x^{1/2}} + \frac{3}{5(4x)^2}$$
$$= \frac{2}{5}x^{-3} - x^{-1/2} + \frac{3}{5}(4x)^{-2}$$

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Rewrite $\dfrac{-6x}{(1-3x^2)^2} + \dfrac{1}{\sqrt[3]{x}}$ using negative exponents.

EXAMPLE 6 Rewriting a Fraction as a Sum of Terms

Rewrite each fraction as the sum of three terms.

a. $\dfrac{x^2 - 4x + 8}{2x}$ **b.** $\dfrac{x + 2x^2 + 1}{\sqrt{x}}$

Solution

a. $$\frac{x^2 - 4x + 8}{2x} = \frac{x^2}{2x} - \frac{4x}{2x} + \frac{8}{2x}$$
$$= \frac{x}{2} - 2 + \frac{4}{x}$$

b. $$\frac{x + 2x^2 + 1}{\sqrt{x}} = \frac{x}{x^{1/2}} + \frac{2x^2}{x^{1/2}} + \frac{1}{x^{1/2}}$$
$$= x^{1/2} + 2x^{3/2} + x^{-1/2}$$

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Rewrite $\dfrac{x^4 - 2x^3 + 5}{x^3}$ as the sum of three terms.

A.7 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: **Fill in the blanks.**

1. To rewrite the expression $\dfrac{3}{x^5}$ using negative exponents, move x^5 to the ________ and change the sign of the exponent.

2. When dividing fractions, multiply by the ________.

Skills and Applications

Describing and Correcting an Error **In Exercises 3–22, describe and correct the error.**

3. $2x - (3y + 4) = 2x - 3y + 4$

4. $5z + 3(x - 2) = 5z + 3x - 2$

5. $\dfrac{4}{16x - (2x + 1)} = \dfrac{4}{14x + 1}$

6. $\dfrac{1 - x}{(5 - x)(-x)} = \dfrac{x - 1}{x(x - 5)}$

7. $(5z)(6z) = 30z$

8. $x(yz) = (xy)(xz)$

9. $a\left(\dfrac{x}{y}\right) = \dfrac{ax}{ay}$

10. $(4x)^2 = 4x^2$

11. $\sqrt{x + 9} = \sqrt{x} + 3$

12. $\sqrt{25 - x^2} = 5 - x$

13. $\dfrac{2x^2 + 1}{5x} = \dfrac{2x + 1}{5}$

14. $\dfrac{6x + y}{6x - y} = \dfrac{x + y}{x - y}$

15. $\dfrac{1}{a^{-1} + b^{-1}} = \left(\dfrac{1}{a + b}\right)^{-1}$

16. $\dfrac{1}{x + y^{-1}} = \dfrac{y}{x + 1}$

17. $(x^2 + 5x)^{1/2} = x(x + 5)^{1/2}$

18. $x(2x - 1)^2 = (2x^2 - x)^2$

19. $\dfrac{3}{x} + \dfrac{4}{y} = \dfrac{7}{x + y}$

20. $\dfrac{1}{2y} = (1/2)y$

21. $\dfrac{x}{2y} + \dfrac{y}{3} = \dfrac{x + y}{2y + 3}$

22. $5 + (1/y) = \dfrac{1}{5 + y}$

Inserting Factors in an Expression **In Exercises 23–44, insert the required factor in the parentheses.**

23. $\dfrac{5x + 3}{4} = \dfrac{1}{4}\left(\boxed{\quad}\right)$

24. $\dfrac{7x^2}{10} = \dfrac{7}{10}\left(\boxed{\quad}\right)$

25. $\frac{2}{3}x^2 + \frac{1}{3}x + 5 = \frac{1}{3}\left(\boxed{\quad}\right)$

26. $\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}\left(\boxed{\quad}\right)$

27. $x^2(x^3 - 1)^4 = \left(\boxed{\quad}\right)(x^3 - 1)^4(3x^2)$

28. $x(1 - 2x^2)^3 = \left(\boxed{\quad}\right)(1 - 2x^2)^3(-4x)$

29. $2(y - 5)^{1/2} + y(y - 5)^{-1/2} = (y - 5)^{-1/2}\left(\boxed{\quad}\right)$

30. $3t(6t + 1)^{-1/2} + (6t + 1)^{1/2} = (6t + 1)^{-1/2}\left(\boxed{\quad}\right)$

31. $\dfrac{4x + 6}{(x^2 + 3x + 7)^3} = \left(\boxed{\quad}\right)\dfrac{1}{(x^2 + 3x + 7)^3}(2x + 3)$

32. $\dfrac{x + 1}{(x^2 + 2x - 3)^2} = \left(\boxed{\quad}\right)\dfrac{1}{(x^2 + 2x - 3)^2}(2x + 2)$

33. $\dfrac{3}{x} + \dfrac{5}{2x^2} - \dfrac{3}{2}x = \left(\boxed{\quad}\right)(6x + 5 - 3x^3)$

34. $\dfrac{(x - 1)^2}{169} + (y + 5)^2 = \dfrac{(x - 1)^3}{169\left(\boxed{\quad}\right)} + (y + 5)^2$

35. $\dfrac{25x^2}{36} + \dfrac{4y^2}{9} = \dfrac{x^2}{\left(\boxed{\quad}\right)} + \dfrac{y^2}{\left(\boxed{\quad}\right)}$

36. $\dfrac{5x^2}{9} - \dfrac{16y^2}{49} = \dfrac{x^2}{\left(\boxed{\quad}\right)} - \dfrac{y^2}{\left(\boxed{\quad}\right)}$

37. $\dfrac{x^2}{3/10} - \dfrac{y^2}{4/5} = \dfrac{10x^2}{\left(\boxed{\quad}\right)} - \dfrac{5y^2}{\left(\boxed{\quad}\right)}$

38. $\dfrac{x^2}{5/8} + \dfrac{y^2}{6/11} = \dfrac{8x^2}{\left(\boxed{\quad}\right)} + \dfrac{11y^2}{\left(\boxed{\quad}\right)}$

39. $x^{1/3} - 5x^{4/3} = x^{1/3}\left(\boxed{\quad}\right)$

40. $3(2x + 1)x^{1/2} + 4x^{3/2} = x^{1/2}\left(\boxed{\quad}\right)$

41. $(1 - 3x)^{4/3} - 4x(1 - 3x)^{1/3} = (1 - 3x)^{1/3}\left(\boxed{\quad}\right)$

42. $\dfrac{1}{2\sqrt{x}} + 5x^{3/2} - 10x^{5/2} = \dfrac{1}{2\sqrt{x}}\left(\boxed{\quad}\right)$

43. $\dfrac{1}{10}(2x + 1)^{5/2} - \dfrac{1}{6}(2x + 1)^{3/2} = \dfrac{(2x + 1)^{3/2}}{15}\left(\boxed{\quad}\right)$

44. $\dfrac{3}{7}(t + 1)^{7/3} - \dfrac{3}{4}(t + 1)^{4/3} = \dfrac{3(t + 1)^{4/3}}{28}\left(\boxed{\quad}\right)$

Rewriting with Negative Exponents **In Exercises 45–50, rewrite the expression using negative exponents.**

45. $\dfrac{7}{(x + 3)^5}$

46. $\dfrac{2 - x}{(x + 1)^{3/2}}$

47. $\dfrac{2x^5}{(3x + 5)^4}$

48. $\dfrac{x + 1}{x(6 - x)^{1/2}}$

49. $\dfrac{4}{3x} + \dfrac{4}{x^4} - \dfrac{7x}{\sqrt[3]{2x}}$

50. $\dfrac{x}{x - 2} + \dfrac{1}{x^2} + \dfrac{8}{3(9x)^3}$

Rewriting a Fraction as a Sum of Terms **In Exercises 51–56, rewrite the fraction as a sum of two or more terms.**

51. $\dfrac{x^2 + 6x + 12}{3x}$

52. $\dfrac{x^3 - 5x^2 + 4}{x^2}$

53. $\dfrac{4x^3 - 7x^2 + 1}{x^{1/3}}$

54. $\dfrac{2x^5 - 3x^3 + 5x - 1}{x^{3/2}}$

55. $\dfrac{3 - 5x^2 - x^4}{\sqrt{x}}$

56. $\dfrac{x^3 - 5x^4}{3x^2}$

Simplifying an Expression In Exercises 57–68, simplify the expression.

57. $\dfrac{-2(x^2-3)^{-3}(2x)(x+1)^3-3(x+1)^2(x^2-3)^{-2}}{[(x+1)^3]^2}$

58. $\dfrac{x^5(-3)(x^2+1)^{-4}(2x)-(x^2+1)^{-3}(5)x^4}{(x^5)^2}$

59. $\dfrac{(6x+1)^3(27x^2+2)-(9x^3+2x)(3)(6x+1)^2(6)}{[(6x+1)^3]^2}$

60. $\dfrac{(4x^2+9)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2}$

61. $\dfrac{(x+2)^{3/4}(x+3)^{-2/3}-(x+3)^{1/3}(x+2)^{-1/4}}{[(x+2)^{3/4}]^2}$

62. $(2x-1)^{1/2}-(x+2)(2x-1)^{-1/2}$

63. $\dfrac{2(3x-1)^{1/3}-(2x+1)\left(\frac{1}{3}\right)(3x-1)^{-2/3}(3)}{(3x-1)^{2/3}}$

64. $\dfrac{(x+1)\left(\frac{1}{2}\right)(2x-3x^2)^{-1/2}(2-6x)-(2x-3x^2)^{1/2}}{(x+1)^2}$

65. $\dfrac{1}{(x^2+4)^{1/2}}\cdot\dfrac{1}{2}(x^2+4)^{-1/2}(2x)$

66. $\dfrac{1}{x^2-6}(2x)+\dfrac{1}{2x+5}(2)$

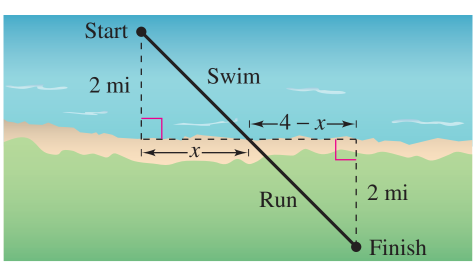
67. $(x^2+5)^{1/2}\left(\frac{3}{2}\right)(3x-2)^{1/2}(3)$
$\quad + (3x-2)^{3/2}\left(\frac{1}{2}\right)(x^2+5)^{-1/2}(2x)$

68. $(3x+2)^{-1/2}(3)(x-6)^{1/2}(1)$
$\quad + (x-6)^3\left(-\frac{1}{2}\right)(3x+2)^{-3/2}(3)$

69. **Athletics** An athlete has set up a course for training as part of her regimen in preparation for an upcoming triathlon. She is dropped off by a boat 2 miles from the nearest point on shore. The finish line is 4 miles down the coast and 2 miles inland (see figure). She can swim 2 miles per hour and run 6 miles per hour. The time t (in hours) required for her to reach the finish line can be approximated by the model

$$t = \frac{\sqrt{x^2+4}}{2} + \frac{\sqrt{(4-x)^2+4}}{6}$$

where x is the distance down the coast (in miles) to the point at which she swims and then leaves the water to start her run.



(a) Find the times required for the triathlete to finish when she swims to the points $x = 0.5$, $x = 1.0, \ldots, x = 3.5$, and $x = 4.0$ miles down the coast.

(b) Use your results from part (a) to determine the distance down the coast that will yield the minimum amount of time required for the triathlete to reach the finish line.

(c) The expression below was obtained using calculus. It can be used to find the minimum amount of time required for the triathlete to reach the finish line. Simplify the expression.

$$\tfrac{1}{2}x(x^2+4)^{-1/2} + \tfrac{1}{6}(x-4)(x^2-8x+20)^{-1/2}$$

70. **Verifying an Equation**

(a) Verify that $y_1 = y_2$ analytically.

$$y_1 = x^2\left(\frac{1}{3}\right)(x^2+1)^{-2/3}(2x) + (x^2+1)^{1/3}(2x)$$

$$y_2 = \frac{2x(4x^2+3)}{3(x^2+1)^{2/3}}$$

(b) Complete the table and demonstrate the equality in part (a) numerically.

x	-2	-1	$-\frac{1}{2}$	0	1	2	$\frac{5}{2}$
y_1							
y_2							

Exploration

71. **Writing** Write a paragraph explaining to a classmate why $\dfrac{1}{(x-2)^{1/2}+x^4} \neq (x-2)^{-1/2}+x^{-4}$.

72. **Think About It** You are taking a course in calculus, and for one of the homework problems you obtain the following answer.

$$\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2}$$

The answer in the back of the book is

$$\frac{1}{15}(2x-1)^{3/2}(3x+1).$$

Show how the second answer can be obtained from the first. Then use the same technique to simplify each of the following expressions.

(a) $\dfrac{2}{3}x(2x-3)^{3/2} - \dfrac{2}{15}(2x-3)^{5/2}$

(b) $\dfrac{2}{3}x(4+x)^{3/2} - \dfrac{2}{15}(4+x)^{5/2}$